Symmetric Information Rate for Continuous Phase Channel and BLAST Architecture with CPM MIMO System

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***Abstract*— How to calculate the symmetric information rate (SIR) for continuous phase channels is given. This problem has traditionally been viewed highly complicated due to nonlinearity of the modulation. Through the simulated SIR it can be observed that using continuous phase modulation (CPM) in a multiple antenna system (MIMO) will likely produce both high performance and high throughput. As the optimum CPM MIMO demodulation has a significant complexity, the BLAST architecture is applied and simulation results are provided.**

I. Introduction

Given a particular channel, a first thing is to determine its achievable transmission rate. In a typical continuous channel model such as in AWGN channel, the input and output are each a set of time functions, and for each input waveform the output is a random process. The probability of a random waveform is often much more involved than that of a random variable. In [1] Gallager takes the approach to represent waveform as a series expansion of a set of orthonormal waveforms (i.e., Karhunen-Loeve expansion), then characterizes the random waveform in terms of joint probability distributions over coefficients in such a series expansion. With linear modulation, this approach results in the discrete outputs after matched filtering. Hence the continuous waveform channel is transformed into an equivalent discrete channel and the difficulty associated with the probability of a random process is bypassed.

For system with input from finite alphabet size, symmetric information rate (SIR) is the measure often considered in place of Shannon capacity. SIR is the achievable information rate with i.i.d. equiprobable inputs. It can serve as a lower bound on the capacity constrained on the modulation format. The problem of calculating the SIR for intersymbol interference (ISI) channel has been considered in [2,3]. Hirt [2] develops a Monte Carlo method for estimating the SIR with the complexity being proportional to M^{N_c} where N_c is the input block length and M is the input alphabet size. In [3] Shamai *et. al.* presents several bounds formulated in terms of memoryless channel with i.i.d. inputs. Recently, how to calculate the SIR for ISI channel based on BCJR algorithm [4] has been independently reported in [5–7]. This method is again simulation-based, but its complexity is only proportional to M^{ν} where ν is the memory length of the channel. The major contribution of these works is the recursive computation of the output entropy rate.

In the case where large transmitted power (i.e., long range) is desired linear modulations have the deficiency that the output envelope varies over time. This variation in the envelope and the nonlinearity of a typical high power amplifier in situations where the spectral mask is tightly specified imply that an expensive linear amplifier is necessary or that the transmitted power must be significantly backed off. On the other hand, continuous phase modulation (CPM) has been attractive for digital transmission over both power and bandwidth limited communication links [8,9]. The constant envelope of CPM signals makes it possible to use low-cost, nonlinear power amplifiers. Meanwhile, CPM maintains phase continuity through an introduced memory to achieve bandwidth efficiency. The effect of this memory is to produce a more complex optimum demodulator.

How to calculate the SIR of CPM signals over continuous channel is a topic, we believe, never been addressed before. The main obstacle is that with nonlinear modulation, it is hard to come up with a set of orthonormal basis which can give a simplified but equivalent discrete channel model. Fortunately, since CPM can be viewed as a traditional coding process we are able to extend the method with linear modulation in [5–7] to continuous phase channel without resorting to the discrete model.

Although CPM has significant advantages for the low power, low cost transmitters, typically the lower bandwidth efficiency has precluded its widespread use due to the inability to communication information through amplitude. Since the multiple antenna processing establishes a high rate system by implementing many low rate parallel channels, using CPM in a multiple antenna (MIMO) system will likely achieve both high performance and high throughput.

The rest of this paper is organized as follows. Section II gives the calculation of SIR with CPM signals over both AWGN and fading channel. In section III the linear processing of CPM MIMO system is performed. Simulation results are presented in each section respectively and section IV concludes.

II. SIR for Continuous Phase Channel

In this section, a technique to calculate SIR for CPM with AWGN is presented, then the results are extended to fading

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environment. With AWGN, the received CPM signal[1] has the form

$$Y(t) = X(t, \vec{D}(N_c - 1)) + N(t) \quad (1)$$

$$X(t, \vec{D}(N_c - 1)) = \sqrt{\frac{E_s}{T}} \exp\left(j\phi(t, \vec{D}(N_c - 1))\right) \quad (2)$$

where $N(t)$ is a Gaussian random process with the single-sided power spectral density being N_0, $\vec{D}(N_c - 1)$ is the input information sequence, N_c is the input block size and E_s is the symbol energy. With appropriate choice of modulation index $h = 2m/p$ where m and p are co-prime integers, $\phi(t, \vec{D}(N_c - 1))$ over each symbol time is constrained to lie on a trellis,

$$\phi(t, \vec{D}(N_c - 1) = \Theta_i(k) + 2\pi h \sum_{l=k-L_q+1}^{k} D(l) q(t - lT)$$
$$kT \le t < (k+1)T \quad (3)$$

where $\Theta_i(k) \pmod{2\pi} \in \Omega_\Theta$ and

$$\Omega_\Theta = \left\{ 0, \frac{2\pi}{p}, \frac{4\pi}{p} \cdots \frac{2(p-1)\pi}{p} \right\} \quad (4)$$

The state at time kT of CPM is defined as

$$\sigma(k) = [\Theta(k),\ D(k - L_q + 1),\ \cdots,\ D(k-1)] \quad (5)$$

$\sigma(k) \in \Omega_\sigma$ and $\| \Omega_\sigma \| = pM^{L_q - 1}$ where $\| \Omega_\sigma \|$ denotes the cardinality of Ω_σ.

The SIR is the mutual information between the input waveform $X(t, \vec{D}(N_c - 1))$ and the output waveform $Y(t)$ when N_c goes to infinity, i.e.,

$$SIR = I(\mathcal{X}, \mathcal{Y}) = \lim_{N_c \to \infty} \frac{1}{N_c} I(X(t, \vec{D}(N_c - 1)), Y(t)) \quad (6)$$

For a stationary ergodic random processes the Shannon-McMillan-Breiman theorem [10] states that

$$H(\mathcal{Y}) \longrightarrow - \lim_{N_c \to \infty} \frac{1}{N_c} \log [p(y(t))] \quad (7)$$

with probability one. Consequently, an estimate of output entropy rate can be obtained by generating a long realization, $y(t)$, and computing $\log [p(y(t))]$. This can be implemented by observing that $p(y(t))$ can be computed recursively as in the BCJR algorithm [5–7]. More specifically, using standard results from detection theory [11] we can show that

$$f\{y(k)|\sigma, d(k)\} = \quad (8)$$
$$C \exp\{- \frac{\int_{kT}^{(k+1)T} |y(t) - x(t, \sigma, d(k))|^2 dt}{N_0}\}$$

[1]To clarify our notation, we use capital letters to represent the random quantities and lower case letters to represent their realizations.

where $y(k) = \{y(t),\ kT \le t < (k+1)T\}$ and C is a constant. Similarly as in BCJR, we define

$$\alpha_k(s) = \Pr(\sigma(k) = s | \vec{y}(k-1)) \quad (9)$$

where $\vec{y}(k-1) = \{y(k-1),\ \cdots,\ y(0)\}$.

The forward recursion to compute $\alpha_k(s)$ is:

$$\alpha_k(s) = \sum_{s' \in \Omega_\sigma} \alpha_{k-1}(s') \Pr(\sigma(k) = s | \sigma(k-1) = s', \vec{y}(k-1)) \quad (10)$$

The last term in the above summation (10) can be given as

$$\Pr(S_k = s | S_{k-1} = s', y(k-1))$$
$$= N_\alpha \Pr(y(k-1) | \sigma(k-1) = s', \sigma(k) = s) \quad (11)$$
$$= N_\alpha \Pr(y(k-1) | \sigma(k-1) = s', d(k-1)) \quad (12)$$

where N_α is a normalization factor.

Observing that

$$\Pr(\vec{y}(N_c - 1)) = \prod_{k=0}^{N_c - 1} \Pr(y(k) | \vec{y}(k-1)) \quad (13)$$

$$\log \Pr(\vec{y}(N_c - 1)) = \sum_{k=0}^{N_c - 1} \log \Pr(y(k) | \vec{y}(k-1)) \quad (14)$$

$$\Pr(y(k) | \vec{y}(k-1)) = \sum_{s \in \Omega_S} \alpha_k(s) \Pr(y(k) | S_k = s) \quad (15)$$

$$\Pr(y(k) | S_k = s) = \frac{1}{M} \sum_{d(k)} \Pr(y(k) | S_k = s, d(k)) \quad (16)$$

hence the calculation of $\log \Pr(\vec{y}(N_c - 1))$ is fairly straightforward. Since we are only concerned with mutual information, we can normalize equation (16) with respect to equation (8). With this normalization, the constant term C can be cancelled, the conditional entropy $H(\mathcal{Y}|\mathcal{X}) = 0$ and the problem reduces to the computation of $H(\mathcal{Y})$.

The SIR's for uniform binary input with BPSK and 3RC ($h = 2/3$) over AWGN channel are shown in Fig. 1. To make a fair comparison we also provide the power spectral density of 3RC together with GMSK mask in Fig. 2. It can be observed that this particular CPM format produces a larger SIR than BPSK with high spectral efficiency. The reason accounting for this is that with the inherent memory in the phase of CPM signal, it can be designed to yield larger free Euclidean distance while still meet the required spectrum.

In the fading environment, the outage SIR is of more interest as time independent fading is not a reasonable assumption unless the Doppler spread is sufficiently high. Using the algorithm derived above, we can easily simulate the outage SIR for CPM in quasi-static fading. Fig. 3 illustrates the outage probability vs. rate with different SNR for MSK signaling [2]. There

[2]Throughout this paper, the SNR of all figures is normalized with respect to the number of transmit antennas.

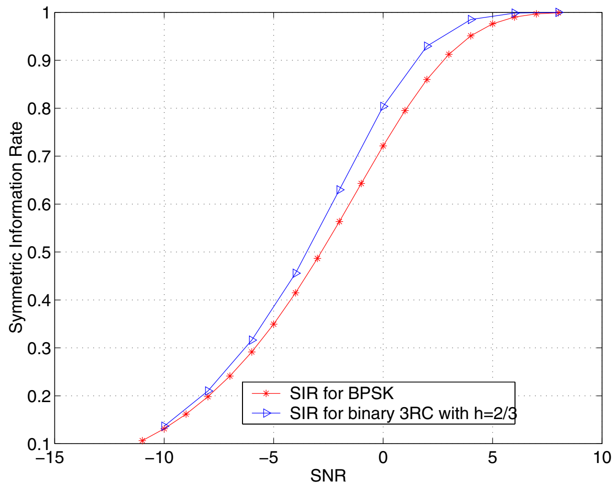

Fig. 1. Simulated SIR for BPSK and binary 3RC with $h = 2/3$

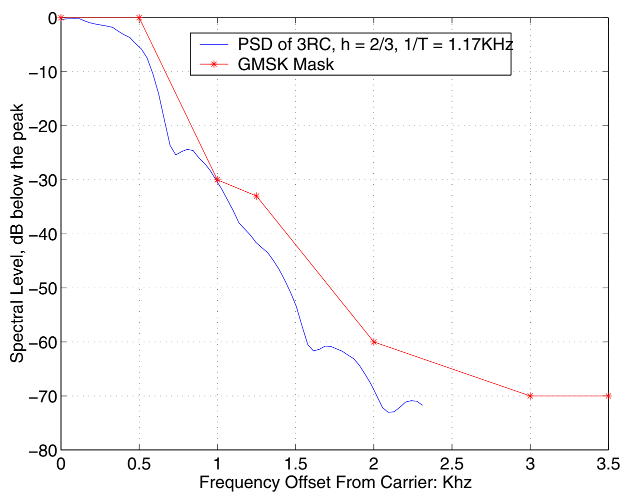

Fig. 2. Power Spectral Density for 3RC with $h = 2/3$

are 1 transmit antenna and 2 receive antennas available in this system. Different fading path is assumed to be mutually independent. The diversity level promised by this system is 2, which can be told by the same slope of all these curves. On the other hand, with a rate 0.5 bit/second, if we want to achieve an outage probability less than 0.03, the SNR as high as 7dB is required. This figure also indicates that the maximum achievable rate of this system is 1 bit/second.

Fig. 4 demonstrates the outage probability vs. SNR with different rate. There are 2 transmit antennas and 2 receive antennas available. Again from these curves we can observe that the lower the rate, the smaller the outage probability and all the curves have the same slope. Comparing Fig. 4 with Fig. 3 it can be seen that the system with 2 transmit antennas and 2 receive antennas can provide a higher rate than the system with only 1 transmit antenna and 2 receive antennas. With a SNR 7 dB and an outage probability around 0.09, the 2×2 system can provide a rate as high as 1.6 bits/second, while the 1×2 can only transmit the symbol at rate 0.8 bit/second.

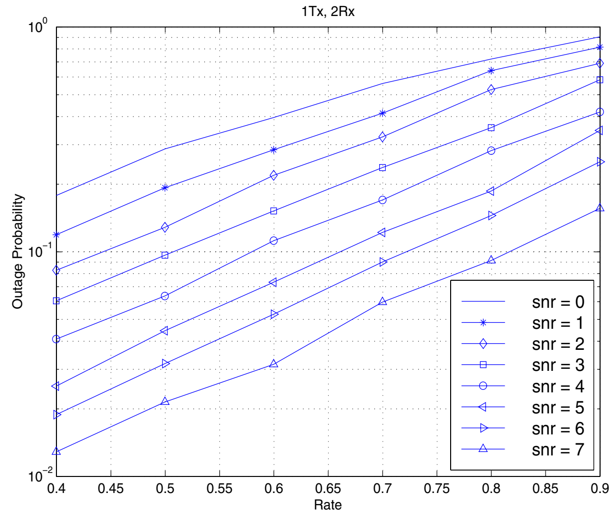

Fig. 3. Outage probability vs. rate with 1 Tx antenna and 2 Rx antennas

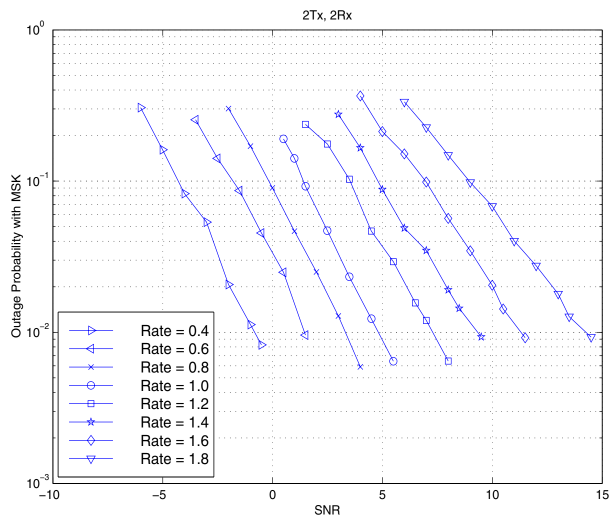

Fig. 4. Outage probability vs. SNR with 2 Tx antennas and 2 Rx antennas

III. CPM MIMO Processing

In theory multiple antenna systems can greatly increase the capacity, as well as significantly lower the probability of error, of a wireless communication links. From the simulations above we can see that CPM can be designed to achieve better information rate than linear modulation. Since the MIMO system can compensate the relatively lower bandwidth efficiency inherent in CPM signals, using CPM in MIMO will likely provide both high data rate and high performance. The block diagram of CPM MIMO system is shown in Fig. 5 where the number of transmit antenna is L_t and number of receive antenna is L_r.

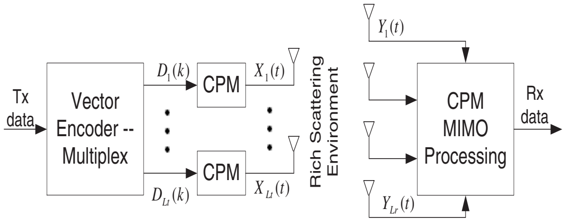

Fig. 5. Block diagram of CPM MIMO system

Optimal CPM MIMO demodulation would have a significant complexity. Maximum likelihood (ML) or maximum *a posteriori* probability (MAP) detection is often too computationally intensive as the complexity grows exponentially with both the number of transmit antennas and the memory length of the particular CPM format. Similarly as in linear modulation, the "divide and conquer" strategy as in V-BLAST architecture [12] can be applied. The idea is rather than jointly decoding the signals from all transmit antennas, a sequential nulling and cancellation scheme is used. The difference is that in V-BLAST, there is no use made of the inner substream coding on each transmit antenna while CPM can not be demodulated with high fidelity without considering the transmitter encoding process. In this paper we propose the decoding method as: derive the strongest CPM waveform from the received signals just by a nulling operation then use a MAP decoder to produce symbol estimates for the strongest antenna. These estimates can be used to cancel out the effect of this corresponding waveform from the received signal and then the second strongest CPM waveform can be detected likewise.

The above mentioned approach can also be applied to compute the SIR. Under the assumption of correct decisions being cancelled out, the CPM MIMO system with either minimum mean square error (MMSE) or zero-forcing (ZF) interference suppression can be decomposed into L_t parallel channels with the effective SNR being

$$\gamma_j = \frac{E_s}{N_0||\vec{w}_j||^2}, \quad j = 1,\ 2,\ \cdots,\ L_t \tag{17}$$

where $\vec{w}_j$ is the nulling vector for the j^{th} channel [12], hence the SIR for each channel can be calculated as in Section II. Note that we also make the Gaussian approximation of the MMSE/ZF filter output like in [13, 14].

Simulation results for CPM MIMO processing with ZF-BLAST, MMSE-BLAST and optimal MAP decoder are provided in the following. The CPM format on each transmit antenna is chosen to be MSK. The system rates are 2bits/second with $L_t = 2, L_r = 2$ and $L_t = 2, L_r = 4$. Optimal MAP decoder is implemented by describing the transmitter via a super-trellis and then performing BCJR decoding. Fig. 6 gives the performance with $L_t = 2$ and $L_r = 2$. For this figure we notice that MMSE-BLAST always outperforms the ZF-BLAST architecture since the MMSE receiver takes the noise into account. But none of them can yield near-optimal performance. In Fig. 7 there are still 2 transmit antennas but 4 receive antennas are present. In this case we observe that the BLAST processing is close to the optimal decoder. The reason is that with more receive antennas more redundancy is available to separate the independent parallel channels. This phenomenon is also reported with linear modulation in [15]. Similar results can be seen with the simulated SIR in Fig. 8 and Fig. 9 where the SIR computed through MMSE-BLAST is closer to MAP with more receive antennas.

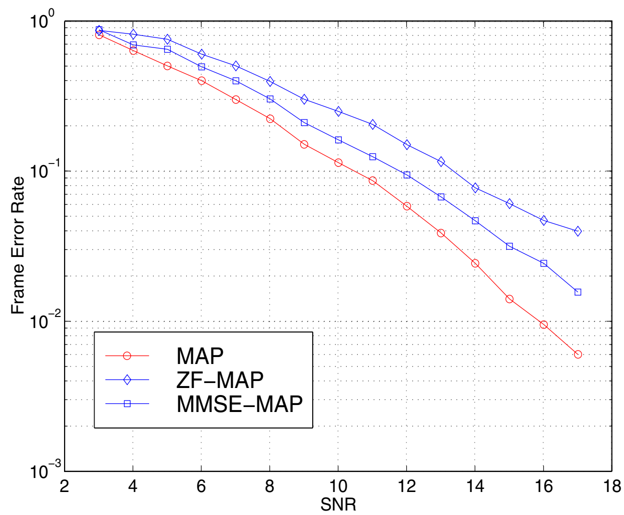

Fig. 6. CPM MIMO performance with $L_t = 2$ and $L_r = 2$

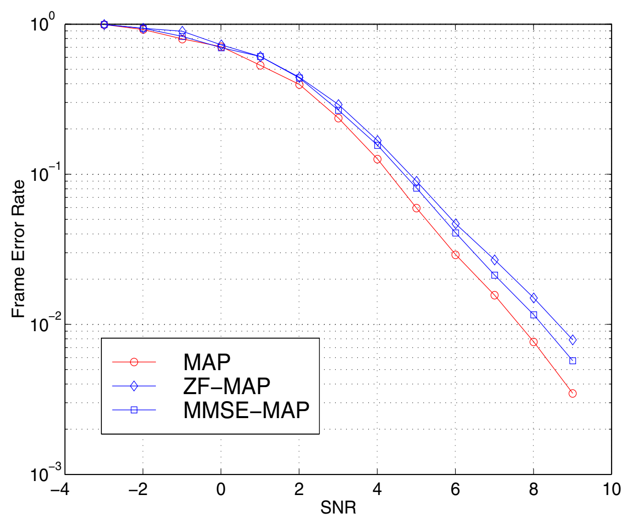

Fig. 7. CPM MIMO performance with $L_t = 2$ and $L_r = 4$

IV. Conclusion

In this paper, we developed the calculation of symmetric information rate for CPM signals. This problem has been tra-

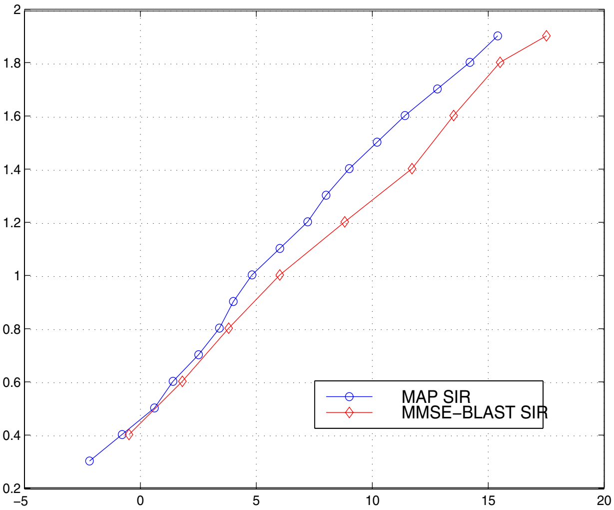

Fig. 8. Symmetric information rate with $L_t = 2$ and $L_r = 2$

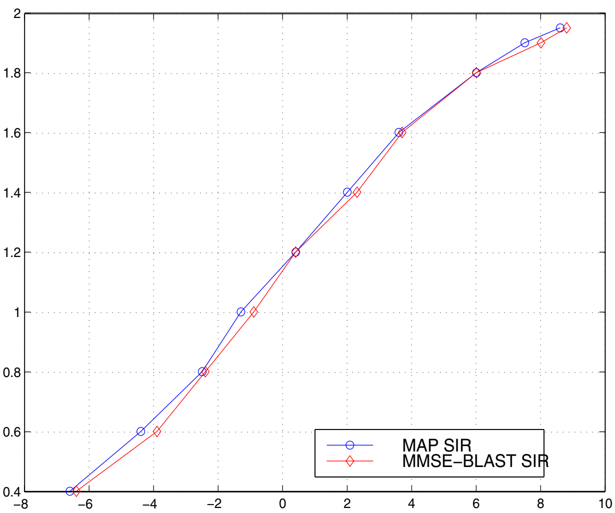

Fig. 9. Symmetric information rate with $L_t = 2$ and $L_r = 4$

ditionally viewed highly complicated due to the nonlinearity of the modulation which makes the equivalent discrete channel model impossible. Through the simulation-based method, the SIR for the CPM channels both in AWGN and in fading environment is provided. The SIR tells that CPM MIMO system can achieve both high data rate and high performance. As the optimal demodulation of CPM MIMO system is often too computationally intensive, suboptimal solution – BLAST architecture combined with MAP decoder for each individual CPM waveform is proposed. Simulation results demonstrate that with more receive antennas than transmit antennas this proposed algorithm can achieve near-optimal performance while the computation complexity is dramatically reduced.

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