

# Space-time Code Design with CPM Transmission <sup>1</sup>

Xiaoxia Zhang  
The Ohio State University  
xiaoxia@ee.eng.ohio-state.edu

Michael P. Fitz  
The Ohio State University  
fitz@ee.eng.ohio-state.edu

**Abstract** — Design criterion for space-time codes with continuous phase modulation (CPM) in quasi-static fading channel is given. Space-time codes are proposed and simulation results will demonstrate their good performance.

## I. INTRODUCTION

Space-time codes have shown considerable promise for reliable transmission over wireless fading channels by efficiently employing transmit diversity [1]. Space-time coding research has to this point focused on linear modulations but continuous phase modulation (CPM) has many desirable characteristics for land mobile wireless communications. This work examines space-time coded CPM and generalizes the previously proposed design rules and provides several optimal codes for CPM.

## II. DESIGN CRITERION

The system considered in this paper is limited to single modulation index CPM, consequently, the complex envelope of the transmitted signal from the  $i^{\text{th}}$  antenna,  $X_i(t)$ ,  $i = 1, \dots, L$ , is given by:

$$X_i(t) = \sqrt{\frac{E_s}{T}} \exp(j\phi(t, \vec{D}_i(k)))$$

$$\phi(t, \vec{D}_i(k)) = 2\pi h \sum_{l=0}^k D_i(l) q_m(t - lT), \quad (k-1)T \leq t < kT$$

where  $D_i(k)$  is the  $k$ th modulation symbol,  $h$  is the modulation index,  $q_m(t)$  is the phase smoothing response function,  $T$  is the symbol time and  $E_s$  is the symbol energy.

Assume a transmission of length  $N_c$  in quasi-static Rayleigh fading, the pairwise error probability (PWE) between two elements of the space-time code, denoted as  $P(\vec{\alpha} \rightarrow \vec{\beta})$ , can be shown to be upper bounded by:

$$P(\alpha \rightarrow \beta) \leq \frac{\left( \begin{matrix} 2\Delta_H(s) - 1 \\ \Delta_H(s) - 1 \end{matrix} \right) N_0^{\Delta_H(s)}}{\prod_{i=1}^{\Delta_H(s)} \lambda_i^{(s)}}$$

where  $\tilde{\lambda}^{(s)}$  are the nonzero eigenvalues of the signal matrix  $C_s$  while  $\Delta_H(s)$  is the number of nonzero eigenvalues. The signal matrix is given as

$$\begin{bmatrix} \int_0^{N_c T} |\Delta_1(t)|^2 dt & \dots & \int_0^{N_c T} \Delta_1^*(t) \Delta_L(t) dt \\ \vdots & \ddots & \vdots \\ \int_0^{N_c T} \Delta_1(t) \Delta_L^*(t) dt & \dots & \int_0^{N_c T} |\Delta_L(t)|^2 dt \end{bmatrix}$$

where

$$\Delta_i(t) = x_i(t, \vec{\alpha}_i) - x_i(t, \vec{\beta}_i)$$

$\Delta_i(t)$  is the continuous time difference between the transmitted signal and the decoded signal on the  $i^{\text{th}}$  antenna. That the integrated continuous time difference between the two codewords is important in nonlinear modulation produces a more difficult design problem than is seen with linear modulation. As with linear modulation it is desirable to achieve a minimum rank of all the signal difference matrices equal to the number of transmitted antennas (especially when a small number of receive antennas is available). This condition will be denoted full space diversity.

In order to achieve full space diversity,  $C_s$  should be full rank. It can be shown that making  $C_s$  full rank is equivalent to making

$$\Delta(t) = a_1 \Delta_1(t) + \dots + a_L \Delta_L(t) \quad (1)$$

nonzero unless  $a_1 = \dots = a_L = 0$  for all  $a_1, \dots, a_L \in \mathcal{C}$

From (1) we can have the following conclusions:

1) Delay diversity schemes always work with CPM transmission as they do with linear modulation.

2) If the number of transmit antennas  $L$  is less than the code alphabet size  $M$ , using different mapping rules over each transmit antenna can give full diversity without involving memory in the space-time code as with linear modulation.

3) Furthermore, if we assume we are equipped with only 2 transmit antennas we can show that as long as every nonzero  $M$ -ary codeword  $\vec{\alpha}$  is a matrix of full rank over the field  $GF(M)$ , then for any CPM transmission full spatial diversity is guaranteed. A straight consequence is that for any code which satisfies the binary rank criterion in [2], it achieves full diversity in CPM transmission.

Given the above observances, we can design good space-time CPM system which is given in Fig. 1. As long as all the

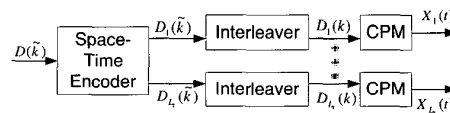


Figure 1: Block diagram of the space-time CPM

interleavers over all the antennas are the same, the diversity level is preserved.

## REFERENCES

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